The extent of variation of the separate spots is very irregular, and does not appear to indicate the operation of any general law. In one or two instances only have neighbouring spots been similarly affected ; thus spots Nos. 5 and 14 in the S.W. quadrant of *Plato*, exhibit the same *decrease* of visibility, and the way in which they have varied from lunation to lunation is somewhat similar, and unlike the manner in which most of the other spots have varied. Spots No. 2 and 18 exhibit the same *increase* of visibility. The great increase of white spots in every part of the Moon's disk, about the time of full, dependent upon the value of ☾ — ☉ would, lunation after lunation, contribute to a steady value of the degree of visibility rather than the irregularity which is indicated by the observations if the *same* spots had been seen month after month. Although the observations amount to 771, and as many as twenty-two spots have been observed on one evening, the average number visible at any given time, as deduced from the 108 series of observations, is seven only, a number which is constant. Upon examining those series in which a smaller number than seven has been recorded, it is found that, besides the spots most commonly seen, viz. Nos. 1, 3, 4, and 17, the remaining two have not been the same. The additional spots seen on these occasions have been very various, several of them having low degrees of visibility, and some of these, which it might be expected could be seen only in the finest weather, have been observed in ordinary states of the Earth's atmosphere.

The observations of the twelve lunations ending in March, 1870, extend considerably the basis on which to found an intelligible explanation of the phenomena, it is, nevertheless, much too narrow to hazard more than conjecture. Another year's observations will doubtless throw further light on the subject.

On the Graphical Construction of the Umbral or Penumbral Curve at any instant during a Solar Eclipse. By Prof. Cayley.

The curve in question, say the penumbral curve, is the intersection of a sphere by a right cone,—I wish to show that the stereographic projection of this curve may be constructed as the envelope of a variable circle, having its centre on a given conic, and cutting at right angles a fixed circle; this fixed circle being in fact the projection of the circle which is the section of the sphere by the plane through the centre and the axis of the cone, or say by the axial plane. The construction thus arrived at is Mr. Casey's construction for a bicircular quartic; and it would not be difficult to show that the stereographic projection of the penumbral curve is in fact a bicircular quartic.

The construction depends on the remark that a right cone is the envelope of a variable sphere having its centre on a given line, and its radius proportional to the distance of the centre

from a given point on this line; and on the following theorem of plane geometry:

Imagine a fixed circle, and a variable circle having its centre on a given line, and its radius proportional to the distance of the centre from a given point on the line (or, what is the same thing, the variable circle always touches a given line); then the locus of the pole in regard to the fixed circle, of the common chord of the two circles (or, what is the same thing, the locus of the centre of a new variable circle which cuts the fixed circle at right angles in the points where it is met by the first-mentioned variable circle) is a conic.

To fix the ideas, say that P is the centre of the first variable circle; AB its common chord with the fixed circle; Q the centre of the circle which cuts the fixed circle at right angles in the points A and B; then the locus of Q is a conic.

To prove this, take $x^2 + y^2 = 1$ for the equation of the fixed circle $(x - \alpha)^2 + (y - \beta)^2 = \gamma^2$ for that of the variable circle; the foregoing law of variation being in fact such that α, β, γ, are linear functions of a variable parameter θ; the equation of the common chord AB is $-2\alpha x - 2\beta y + 1 + \alpha^2 + \beta^2 - \gamma^2 = 0$; viz., this equation contains θ quadratically; hence the envelope of the common chord is a conic; and thence (reciprocating in regard to the fixed circle) the locus of the pole of AB, that is, of the point Q, is also a conic.

Consider now a solid figure in which the circles are replaced by spheres; viz. we have a fixed sphere, and a variable sphere having its centre on a given line and its radius proportional to the distance of the centre from a given point on the line. The envelope of the variable sphere is a right cone; the intersection of the cone with the fixed sphere is the envelope of the small circle of the sphere, say the circle AB, which is the intersection of the fixed sphere by the variable sphere. This circle AB is also the intersection of the fixed sphere by a sphere, centre Q, which cuts the fixed sphere at right angles; and by what precedes the locus of Q is a conic. Hence the penumbral curve is given as the envelope of the circle AB which is the intersection of the fixed sphere by a sphere which has its centre Q on a conic, and which cuts the fixed sphere at right angles. It is obvious that the circle AB always cuts at right angles the great circle which is the section of the fixed sphere by the axial plane, or say the axial circle. Project the whole figure stereographically; the projection of the circle AB is a variable circle which cuts at right angles the circle which is the projection of the axial circle, and which has for its centre the point Q′ which is the projection of Q. But the locus of Q being a conic, the locus of its projection Q′ is also a conic; and we have thus the projection of the penumbral curve as the envelope of a variable circle which has its centre on a conic, and which cuts at right angles a fixed circle.

We may in the axial plane construct five points of the conic which is the locus of Q, by means of any five assumed positions

of the variable circle, and somewhat simplify the construction by a proper choice of the five positions of the variable circle. This is not a convenient construction, and even if it were accomplished we should still have to construct the projection of the conic so obtained, in order to find, in the figure of the stereographic projection, the conic which is the locus of Q′. I do not at present perceive any direct construction for the last-mentioned conic; but assuming that a tolerably simple construction can be obtained, the construction of the projection of the penumbral curve as the envelope of the variable circle is as easy and rapid as possible. Probably the easiest course would be (without using the conic at all) to calculate numerically, for a given position of the variable sphere, the terrestrial latitude and longitude of the two points of intersection of the variable sphere by the axial circle; laying these down on the projection, we have then a position of the variable circle; and a small number of properly selected positions would give the penumbral curve with tolerable accuracy.

I have throughout spoken of the penumbral curve, as it is in regard hereto that a graphical construction is most needed; but the theory is applicable, without any alteration, to the umbral curve.

On the Geometrical Theory of Solar Eclipses.
By Prof. Cayley.

The fundamental equation in a solar eclipse is, I think, most readily established as follows:—

Take the centre of the Earth for origin, and consider a set of axes fixed in the Earth and moveable with it; viz., the axis of z directed towards the North Pole; those of x, y, in the plane of the Equator; the axis of x directed towards the point longitude 0°; that of y towards the point longitude 90° W. of Greenwich. Take a, b, c, for the co-ordinates of the Moon; k for its radius (assuming it to be spherical); a', b', c', for the co-ordinates of the Sun; k' for its radius (assuming it to be spherical); then, writing $\theta + \phi = 1$, the equation

$$\{\theta(x-a)+\phi(x-a')\}^2+\{\theta(y-b)+\phi(y-b')\}^2+\{\theta(z-c)+\phi(z-c')\}^2$$
$$=(\theta k \pm \phi k')^2$$

is the equation of the surface of the Sun or Moon, according as $\theta, \phi = 1, 0$ or $= 0, 1$: and for any values whatever of θ, ϕ, it is that of a variable sphere, such that the whole series of spheres have a common tangent cone. Writing the equation in the form

$$\begin{aligned}&\theta^2\{(x-a)^2+(y-b)^2+(z-c)^2-k^2\}\\ &+2\theta\phi\{(x-a)(x-a')+(y-b)(y-b')+(z-c)(z-c')\mp kk'\}\\ &+\quad\phi^2\{(x-a')^2+(y-b')^2+(z-c')^2-k'^2\}=0,\end{aligned}$$

or, putting for shortness,

$$\begin{aligned} \varrho &= a^2 + b^2 + c^2 - k^2 \\ \varrho' &= a'^2 + b'^2 + c'^2 - k'^2 \\ \sigma &= a\,a' + b\,b' + c\,c' \mp k\,k' \\ \mathrm{P} &= a\,x + b\,y + c\,z \\ \mathrm{P}' &= a'\,x + b\,y + c'\,z, \end{aligned}$$

the equation is

$$\begin{aligned} &\theta^2\ (x^2 + y^2 + z^2 - 2\,\mathrm{P} + \varrho) \\ +\ &2\,\theta\,\varphi\,(x^2 + y^2 + z^2 - \mathrm{P} - \mathrm{P}' + \sigma) \\ +\ &\varphi^2\ (x^2 + y^2 + z^2 - 2\,\mathrm{P}' + \varrho') = 0 \end{aligned}$$

and the equation of the envelope consequently is

$$(x^2 + y^2 + z^2 - 2\,\mathrm{P} + \varrho)\,(x^2 + y^2 + z^2 - 2\,\mathrm{P}' + \varrho') - x^2 + y^2 + z^2 - \mathrm{P} - \mathrm{P}' + \sigma)^2 = 0$$

that is

$$(x^2 + y^2 + z^2)\,(\varrho + \varrho' - 2\,\sigma) - (\mathrm{P} - \mathrm{P}')^2 - 2\,(\varrho' - \sigma)\,\mathrm{P} - 2\,(\varrho - \sigma)\,\mathrm{P}' + \varrho\,\varrho' - \sigma^2 = 0$$

which is the equation of the cone in question.

Observe that one sphere of the series is a *point*, viz., taking first the upper signs, if we have $\theta\,k + \varphi\,k' = 0$, that is

$$\theta = \frac{k'}{k' - k}, \quad \varphi = \frac{-k}{k' - k},$$

then the sphere in question is the point the co-ordinates whereof are

$$x = \frac{k'a - k\,a'}{k' - k}, \quad y = \frac{k'\,b - k\,b'}{k' - k}, \quad z = \frac{k'\,c - k\,c'}{k' - k}$$

which point is the vertex of the cone: it hence appears that, taking the upper signs, the cone is the *umbral* cone, having its vertex on this side of the Moon; and similarly taking the lower signs, then if we have $\theta\,k - \varphi\,k' = 0$, that is

$$\theta = \frac{k'}{k' + k}, \quad \varphi = \frac{k}{k' + k},$$

then the variable sphere will be the point the co-ordinates of which are

$$\frac{k'\,a + k\,a'}{k' + k}, \quad \frac{k'\,b + k\,b'}{k + k}, \quad \frac{k\,c' + k'\,c}{k' + k},$$

which point is the vertex of the cone; viz. the cone is here, the penumbral cone having its vertex between the Sun and Moon.

Taking as unity the Earth's equatorial radius, if p, p' are the

parallaxes, $\varkappa$, $\varkappa'$ the angular semi-diameters of the Moon and Sun respectively, then the distances are $\frac{1}{\sin p}$, $\frac{1}{\sin p'}$ and the radii are $\frac{\sin \varkappa}{\sin p}$, $\frac{\sin \varkappa'}{\sin p'}$ respectively; hence, if h, h' are the hour-angles west from Greenwich, Δ, Δ' the N.P.D.'s of the Moon and Sun respectively, we have

$$a = \frac{1}{\sin p} \sin \Delta \cos h, \quad a' = \frac{1}{\sin p'} \sin \Delta' \cos h',$$

$$b = \frac{1}{\sin p} \sin \Delta \sin h, \quad b' = \frac{1}{\sin p'} \sin \Delta' \sin h',$$

$$c = \frac{1}{\sin p} \cos \Delta \quad , \quad c' = \frac{1}{\sin p'} \cos \Delta',$$

$$k = \frac{\sin \varkappa}{\sin p} \quad , \quad k' = \frac{\sin \varkappa'}{\sin p'}.$$

And thence

$$\varrho = \frac{1}{\sin^2 p} (1 - \sin^2 \varkappa'),$$
$$\varrho' = \frac{1}{\sin^2 p'} (1 - \sin^2 \varkappa'),$$

$$\sigma = \frac{1}{\sin p \sin p'} [\cos \Delta \cos \Delta' + \sin \Delta \sin \Delta' \cos (h' - h) \mp \sin \varkappa \sin \varkappa'],$$

$$P = \frac{1}{\sin p} \{\sin \Delta (x \cos h + y \sin h) + z \cos \Delta\},$$

$$P' = \frac{1}{\sin p'} \{\sin \Delta'(x \cos h' + y \sin h') + z \cos \Delta'\},$$

Moreover, if the right ascensions of the Moon and Sun are α, α' respectively, and if the R.A. of the meridian of Greenwich (or sidereal time in angular measure) be $= \Sigma$, then we have

$$h = \Sigma - \alpha, \quad h' = \Sigma - \alpha'.$$

It is to be observed that $h - h'$, Δ, Δ' are slowly varying quantities, viz., their variation depends upon the variation of the celestial positions of the Sun and Moon; but h and h' depend on the diurnal motion, thus varying about 15° per hour; to put in evidence the rate of variation of the several angles h, h', Δ, Δ' during the continuance of the eclipse, instead of the foregoing values of h, h', I write

$$h' = \left\{ E + \left(1 + \frac{E_1 - E}{24}\right) t \right\} 15^\circ,$$

where t is the Greenwich mean time, E, E_1 are the values (reckoned in parts of an hour) of the Equation of Time at the preceding and following mean noons respectively, taken positively or negatively, so that E, E_1 are the mean times of the two successive apparent noons respectively; whence also

$$h = \left\{E + \left(1 + \frac{E_1 - E}{24}\right) t\right\} 15^\circ - \alpha + \alpha'.$$

And moreover

$$\begin{aligned} \alpha &= A + m\,(t - T), \\ \alpha' &= A + m'(t - T), \\ \Delta &= D + n\,(t - T), \\ \Delta' &= D' + n'\,(t - T), \end{aligned}$$

if T be the time of conjunction, A, A, D, D′ the values at that instant of the R.A.'s and N.P.D.'s; m, m' and n, n' the horary motions in R.A. and N.P.D. respectively.

It appears to me not impossible but that the foregoing form of equation

$$(x^2 + y^2 + z^2)(\varrho + \varrho' - 2\sigma) - (P - P')^2 - 2(\varrho' - \sigma)P - 2(\varrho - \sigma)P' + \varrho\varrho' - \sigma^2 = 0$$

for the umbral or penumbral cone might present some advantage in reference to the calculation of the phenomena of an eclipse over the Earth generally: but in order to obtain in the most simple manner the equation of the same cone referred to a set of principal axes, I proceed as follows :—

Writing

$$\begin{aligned} \mathrm{a} &= bc' - b'c, & \mathrm{f} &= a - a', \\ \mathrm{b} &= ca' - c'a, & \mathrm{g} &= b - b', \\ \mathrm{c} &= ab' - a'b, & \mathrm{h} &= c - c', \end{aligned}$$

(and therefore $\mathrm{af} + \mathrm{bg} + \mathrm{ch} = 0$)

Then, if

$$X = \frac{(\mathrm{bh} - \mathrm{cg})x + (\mathrm{cf} - \mathrm{ah})y + (\mathrm{ag} - \mathrm{bf})z}{\sqrt{\mathrm{a}^2 + \mathrm{b}^2 + \mathrm{c}^2}\,\sqrt{\mathrm{f}^2 + \mathrm{g}^2 + \mathrm{h}^2}},$$

$$Y = \frac{\mathrm{a}x + \mathrm{b}y + \mathrm{c}z}{\sqrt{\mathrm{a}^2 + \mathrm{b}^2 + \mathrm{c}^2}},$$

$$Z = \frac{\mathrm{f}x + \mathrm{g}y + \mathrm{h}z}{\sqrt{\mathrm{f}^2 + \mathrm{g}^2 + \mathrm{h}^2}}.$$

X, Y, Z, will be co-ordinates referring to a new set of rectangular axes; viz., the origin is, as before, at the centre of the Earth, the axis of Z is parallel to the line joining the centres of the Sun and Moon; the axis of X cuts at right angles the last-mentioned line; and the axis of Y is perpendicular to the plane

of the other two axes; or, what is the same thing, to the plane through the centres of the Earth, Sun, and Moon.

The co-ordinates of the vertex of the cone are therefore X_0, Y_0, Z_0, where these denote what the foregoing values of X, Y, Z, become on substituting therein for x, y, z, the values

$$\frac{k' a \mp k a'}{k' \mp k}, \qquad \frac{k' b \mp k b'}{k' \mp k'}, \qquad \frac{k' c \mp k c'}{k' \mp k},$$

and the equation of the cone therefore is

$$(X - X_0)^2 + (Y - Y_0)^2 = \tan^2 \lambda \, (Z - Z_0)^2,$$

where

$$\sin \lambda = \frac{k' \mp k}{G}$$

if for a moment G denotes the distance between the centres of the Sun and Moon. We have therefore

$$\tan \lambda = \frac{k' \mp k}{\sqrt{G^2 - (k' \mp k)^2}},$$

or since

$$G^2 = (a' - a)^2 + (b' - b)^2 + (c' - c)^2,$$

this is in fact

$$\tan \lambda = \frac{k' \mp k}{\sqrt{\varrho + \varrho' - 2\sigma}},$$

where ϱ, ϱ', σ signify as before; and thus X_0, Y_0, Z_0, $\tan \lambda$ are all of them given functions of $a, b, c, k, a', b', c', k'$, and consequently of the before-mentioned astronomical data of the problem. The form is substantially the same as Bessel's equation (3), *Ast. Nach.* No. 321 (1837), (but the direction of the axes of X, Y, is not identical with those of his x, y); and it is therefore unnecessary to consider here the application of it to the calculation of the eclipse for a given point on the Earth.

The Astronomische Gesellschaft.

The third biennial meeting was held at Vienna from the 13th to the 16th of September, 1869, under the presidency of M. Struve; the number of members present was 39; total number 216. The several subjects in question at the meeting of 1867 (*See Monthly Notices*, vol. xxviii. p. 268) were further discussed, and other subjects brought before the meeting; the principal ones were as follows:—

1. As to the New Tables of *Jupiter*, a large part of the